

Using Z-transform Method for AC Analysis

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The idea of using Z-transform technique for AC analysis of switching circuits is not new. For example, Prof. Piero G. Maranesi suggested this approach almost 20 years ago, and even developed circuit simulator FREDOMSIM based on this method. However, FREDOMSIM is not a “true” circuit simulator: it uses simulation data from PSpice to perform Z-transform. What is more important, it is not publicly available (as far as I know). List of publications about FREDOMSIM is available here: <http://nl5.sidelinesoft.com/documents/FREDOMSIM.doc>.

NL5 includes Z-transform method as part of its AC analysis. Along with “Sweep AC source” method, Z-transform is a powerful tool for frequency analysis of switching circuits. It still requires some efforts to make it easy to use and more reliable. However, with understanding of its principles and limitations, the method could give a significant time saving in AC analysis of switching circuits.

Principle of operation.

If switching circuit with switching period T is in “periodic steady state”, all the signals taken at time t and $t+T$ are identical. Adding small “disturbance” to external source, or voltage/current of the component with “memory” (C, L, sample/hold, etc.) at the moment t will result in change of the signals at time $t+T$. This can be expressed by the following simplified formula:

$$X(t+T) = A * X(t) + B * Y(t), \quad (1)$$

where X is vector of circuit signals (state variables, node potentials: whatever is used for circuit analysis), Y is vector of external sources, and A and B are conversion matrices.

Moving from time domain to z-domain, formula (1) can be further converted to:

$$X(z) = H(z) * Y(z), \quad (2)$$

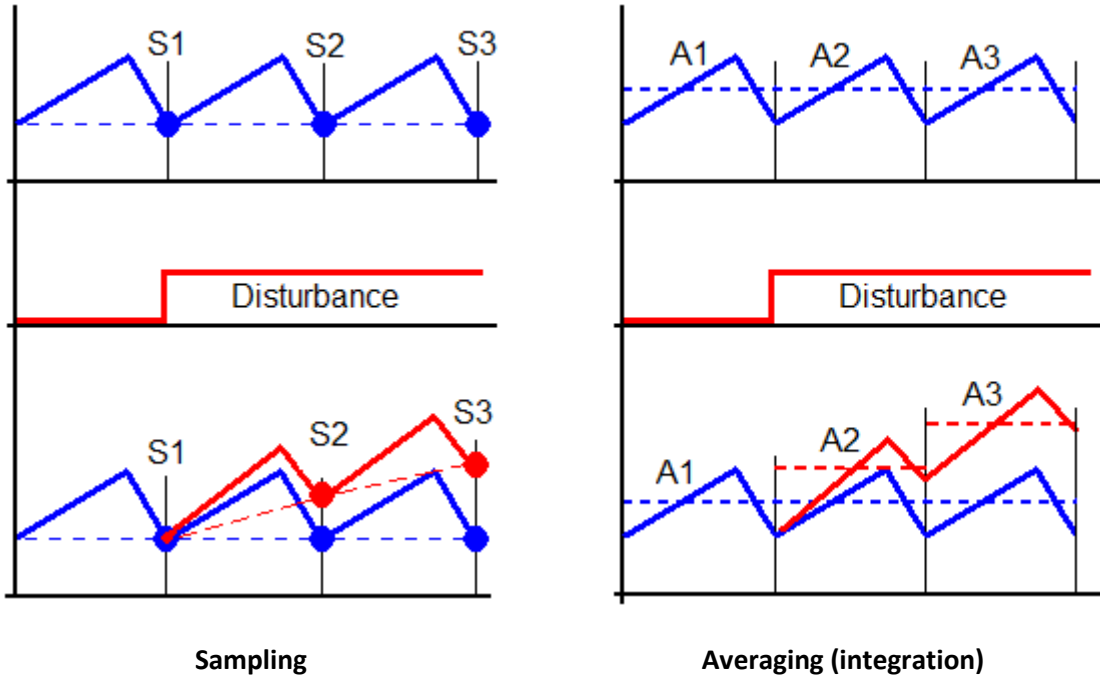
where $z = e^{sT}$, s is Laplace parameter, and $H(z)$ is circuit transfer function in z-domain.

$H(z)$ can be obtained from matrices A and B . Then, substituting $z = e^{sT}$ (or using any other $z \rightarrow s$ conversion method), z-domain transfer function $H(z)$ is converted to s-domain transfer function $H(s)$, or $H(j\omega)$, which is basically AC frequency response of the circuit.

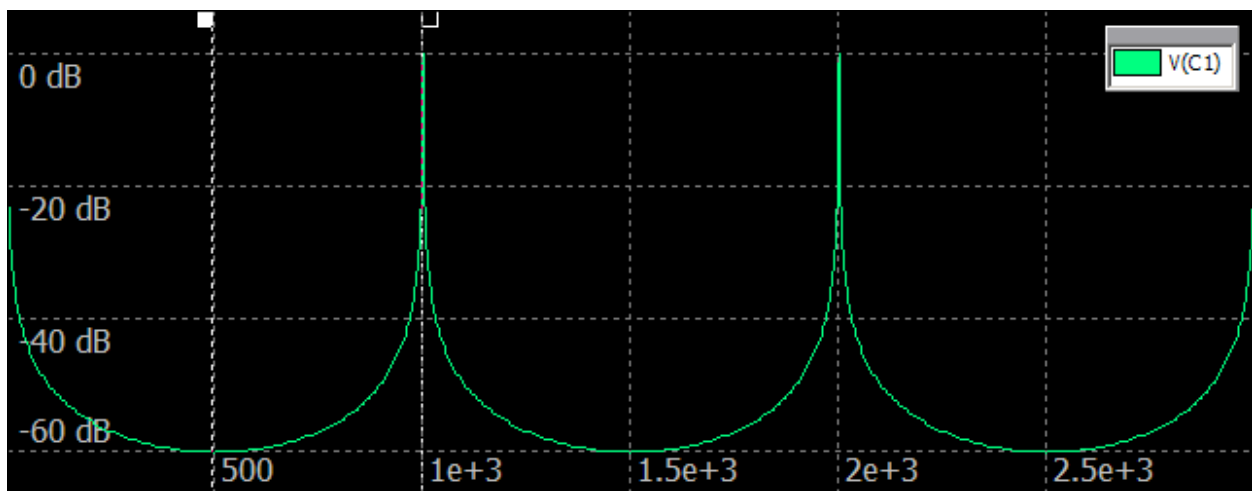
NL5 is capable to obtain matrices A and B by running transient analysis for several switching periods, applying disturbances to the circuit as necessary, and detecting circuit signals at

appropriate time. Then it obtains $H(z)$, and calculates frequency response of the circuit by substituting $z = e^{j\omega T}$ in specified frequency range.

Signals with period T can be sampled at the moments $t, t+T, \dots$, as well as averaged (integrated) over period T :



Since the data points are obtained at discrete time moments $n \cdot T$, the resulting transfer function (spectrum) will be periodic with the period $1/T$:



This is example of an RC-chain transfer function with samples taken at $T = 1$ ms interval. Resulting transfer function is periodic with 1 kHz period. Only results below Nyquist frequency (half of the sampling frequency = 500 Hz) make sense.

Advantages of the method

- Unlike “averaging” technique, the method does not require creating new “averaged” models: it works with exactly the original, non-modified schematic;
- The method is extremely fast: just few switching periods of transient analysis, and then thousands of frequency points calculated with the same speed as simple linear AC analysis.
- User should define just one parameter: switching period, and select couple of algorithm options; everything else is done automatically.

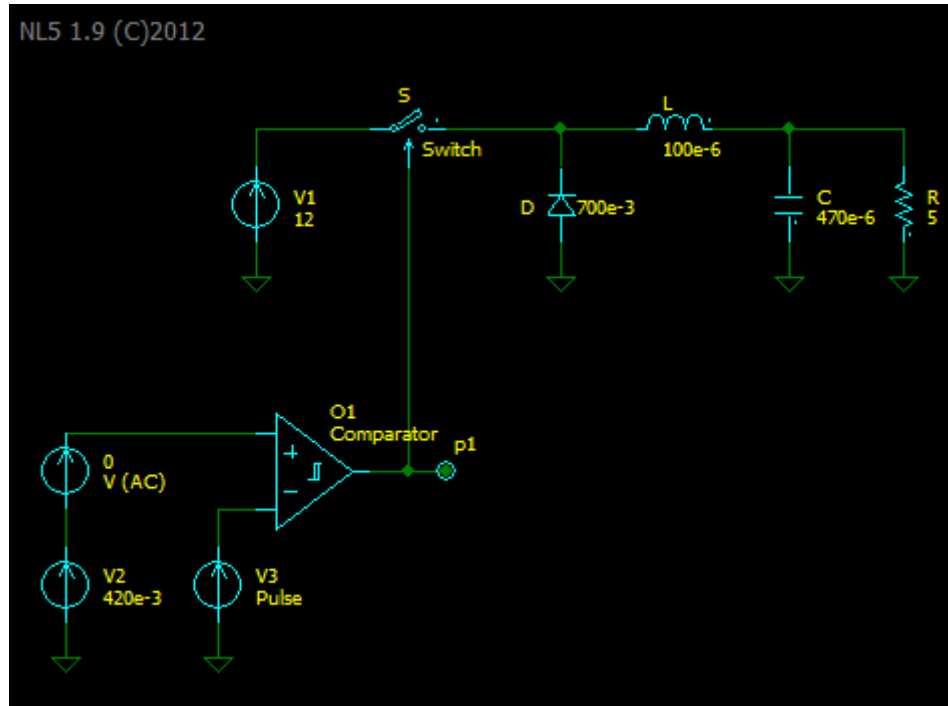
Disadvantages of the method.

- Since the method is using discrete time samples, only results below Nyquist frequency (half of sampling frequency) can be trusted.
- Aliasing may occur for signals with wide spectrum.
- Loss of precision during calculation may cause wrong results at very low frequencies.
- Circuits working in DCM (discontinuous current mode) might have some special problems, which will be addressed below.

To summarize advantages and disadvantages: the method is extremely fast and easy to use, however requires basic knowledge of discrete signal analysis in order to interpret the results properly.

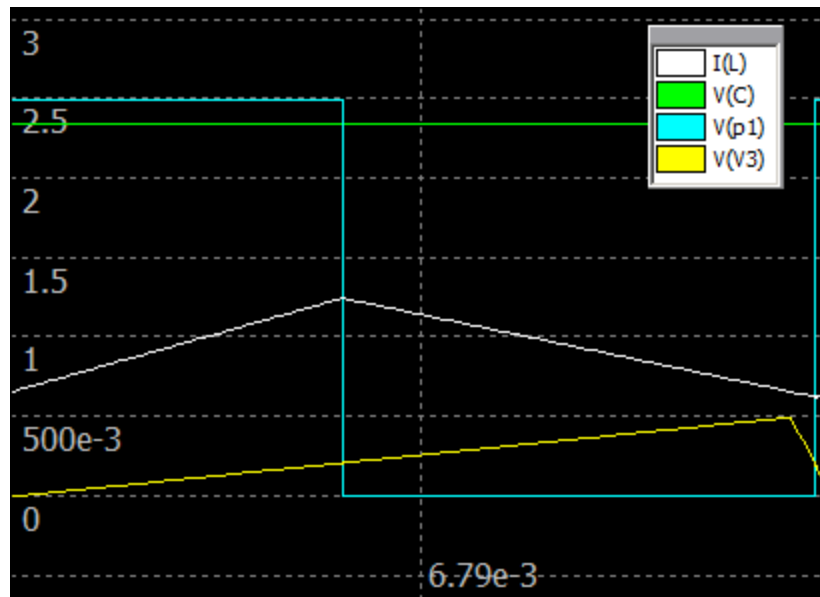
How it works.

Let's see how to use the method on simple buck-converter circuit:



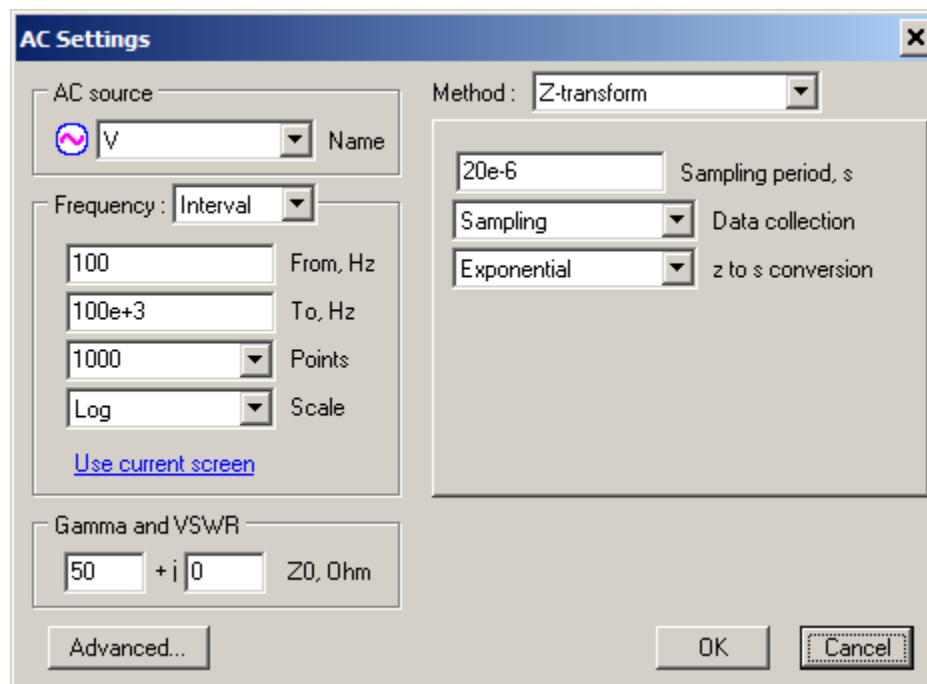
First, the user sets a voltage source, current source, or label which will serve as an input for frequency characteristic. The source should preferably be a constant voltage (current) source, and it should be selected as "AC source".

Second, the user runs transient until the circuit reaches periodic steady state ("periodic operating point"). An easiest way to do that is set screen size equal to the switching period, select "Continuous transient mode", and run transient, watching the traces. Once the picture stabilized, turn off continuous mode, so that transient stops at the end of the screen. The resulting one-period transient snapshot may look like this:

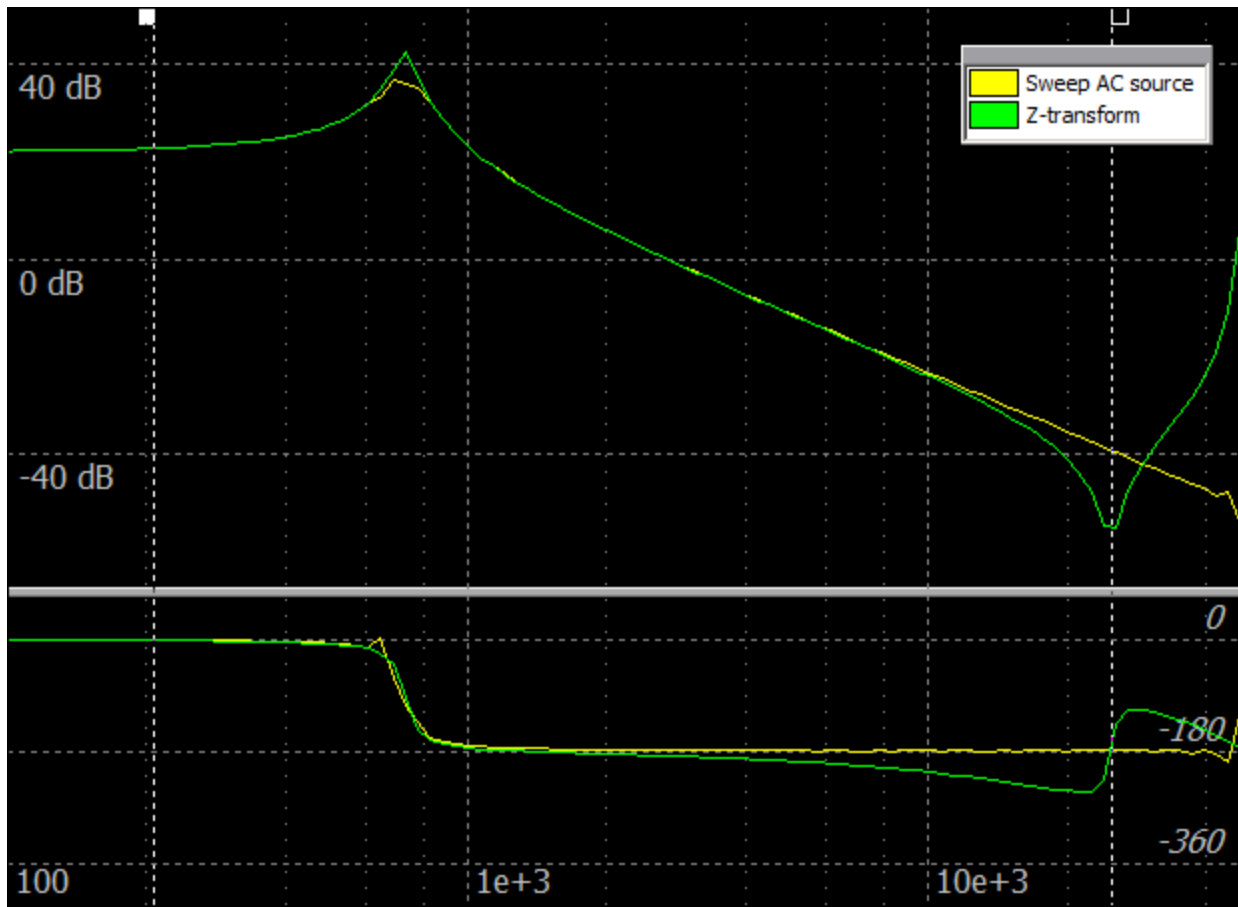


Now the user may save current state to Initial Conditions (Transient/Save IC), so that next time the transient would start being in exactly the same state. However, saving IC is not required. And again: obtaining “periodic steady state” is user’s responsibility.

Next step is setting up “AC settings”:



Just set up required frequency range and number of points, select “Z-transform” method, and set “Sampling period” equal to the circuit switching period. Then close the dialog and start AC analysis. The response will show up in a second:



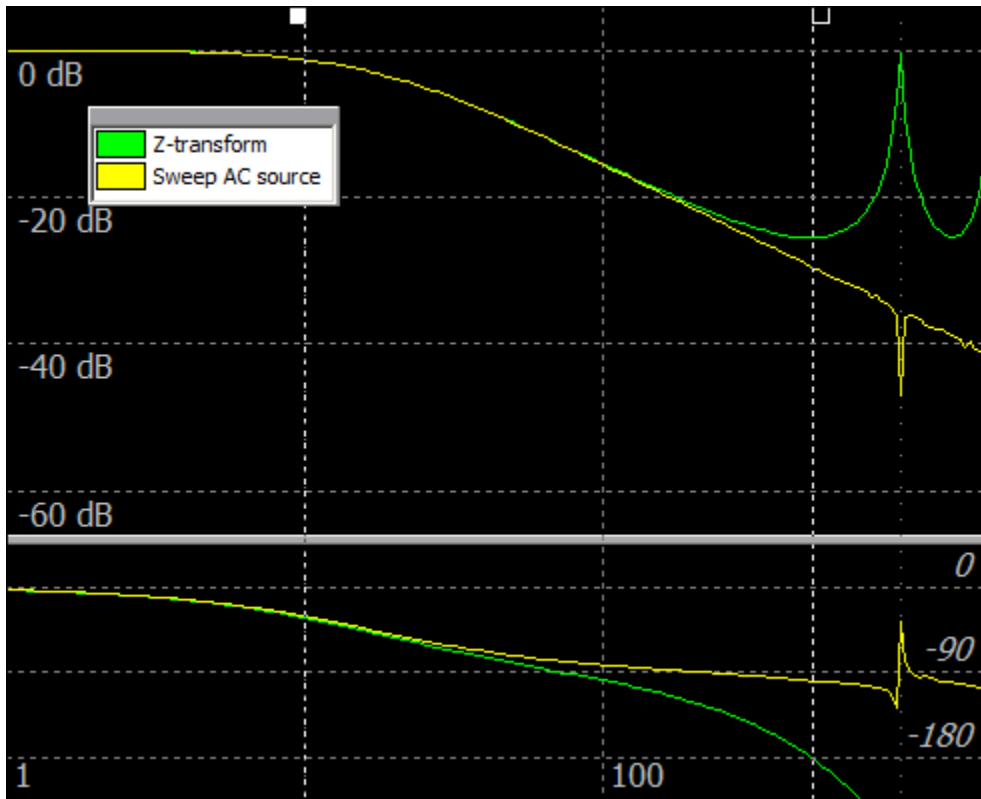
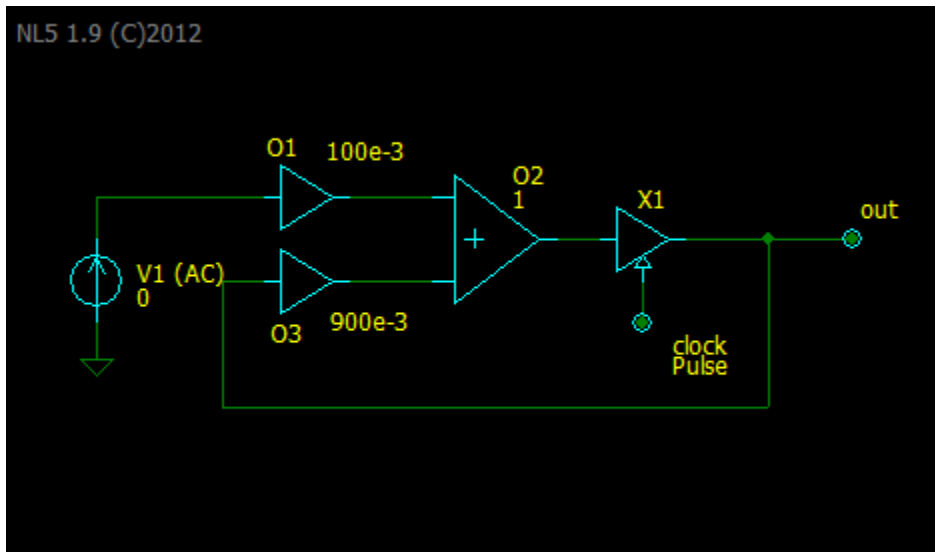
On this graph, the AC response obtained by Z-transform method is compared with “Sweep AC source” (insertion) method. It shows perfect matching at low frequencies, and increased Z-transform distortion close to Nyquist frequency (25 kHz, right cursor).

“Data collection” parameter.

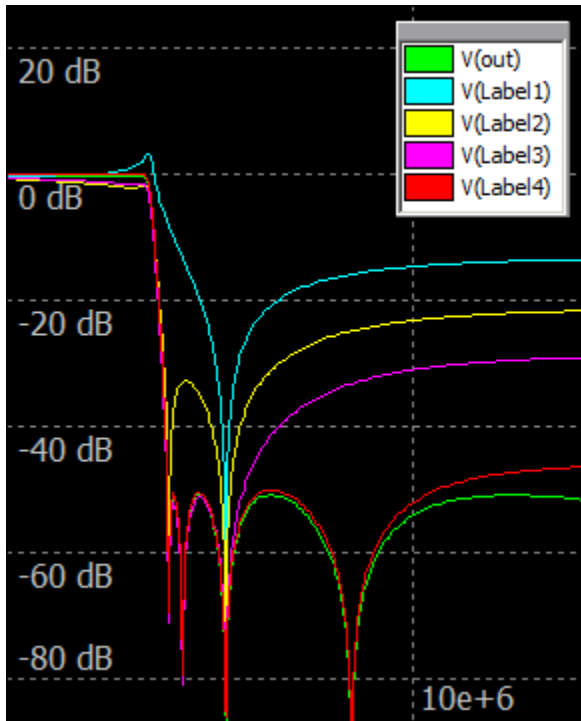
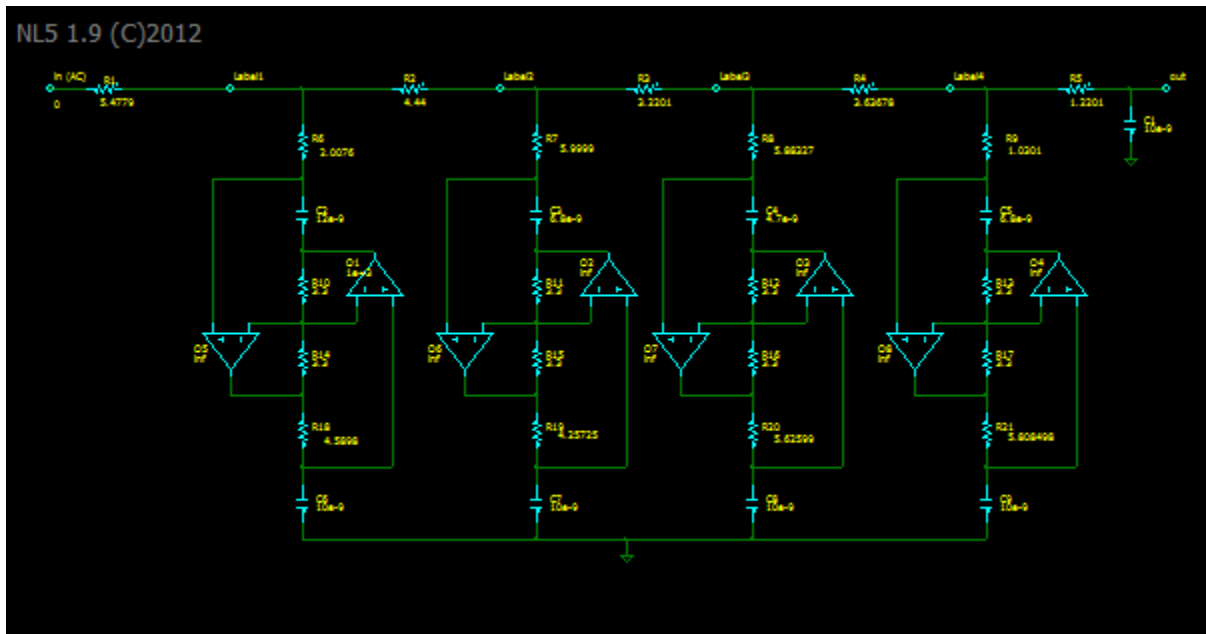
“Data collection” parameter is used to provide reliable and accurate results for different types of circuits and observed signals.

Sampling is a basic option, when all the signals are taken at discrete time t and $t+T$. This is the fastest option, and most accurate at high frequencies (close to Nyquist frequency). It can be used for almost any circuit and observed signals, including switching converters, digital filters (with sample/hold component as “zero-order hold” element), and even linear circuits (I don’t know who would analyze linear circuits with Z-transform, however it clearly confirms validity of the method). Some examples:

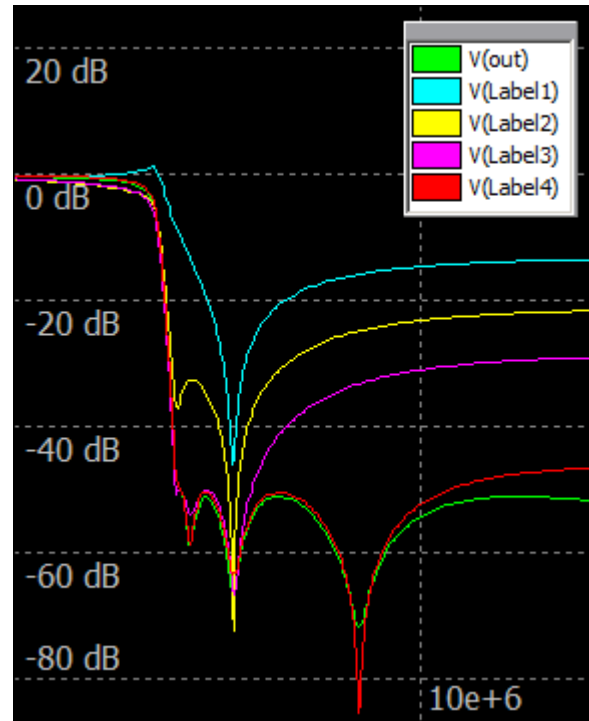
Digital filter.



Linear circuit: Cauer filter:



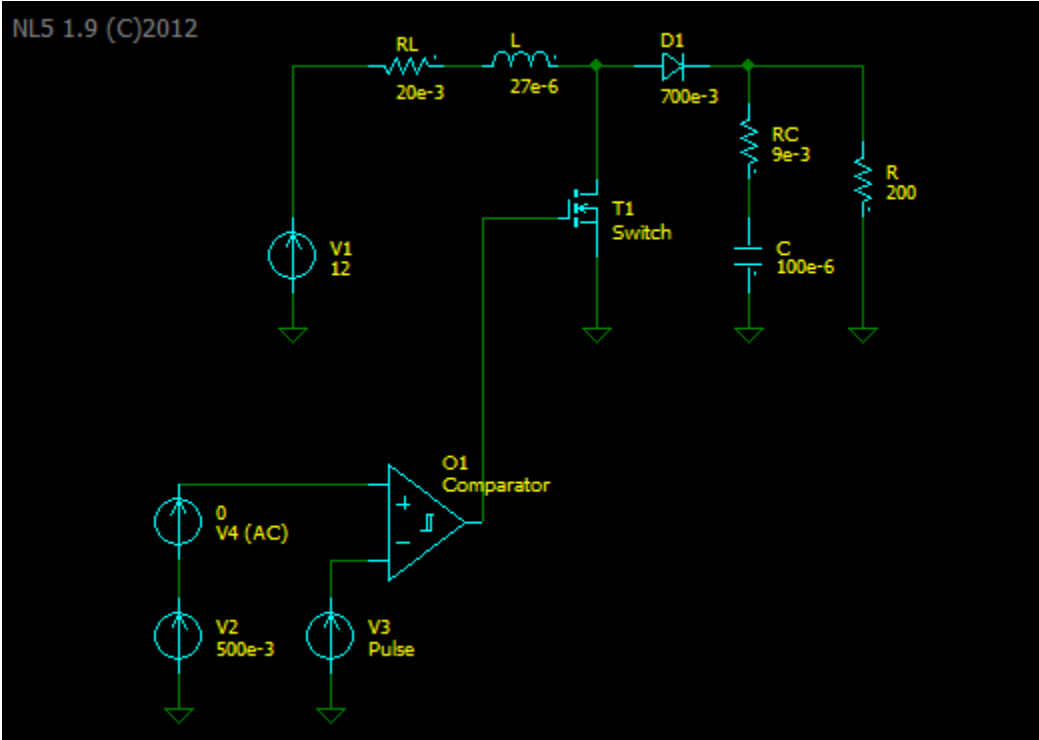
"Linearize schematic" method



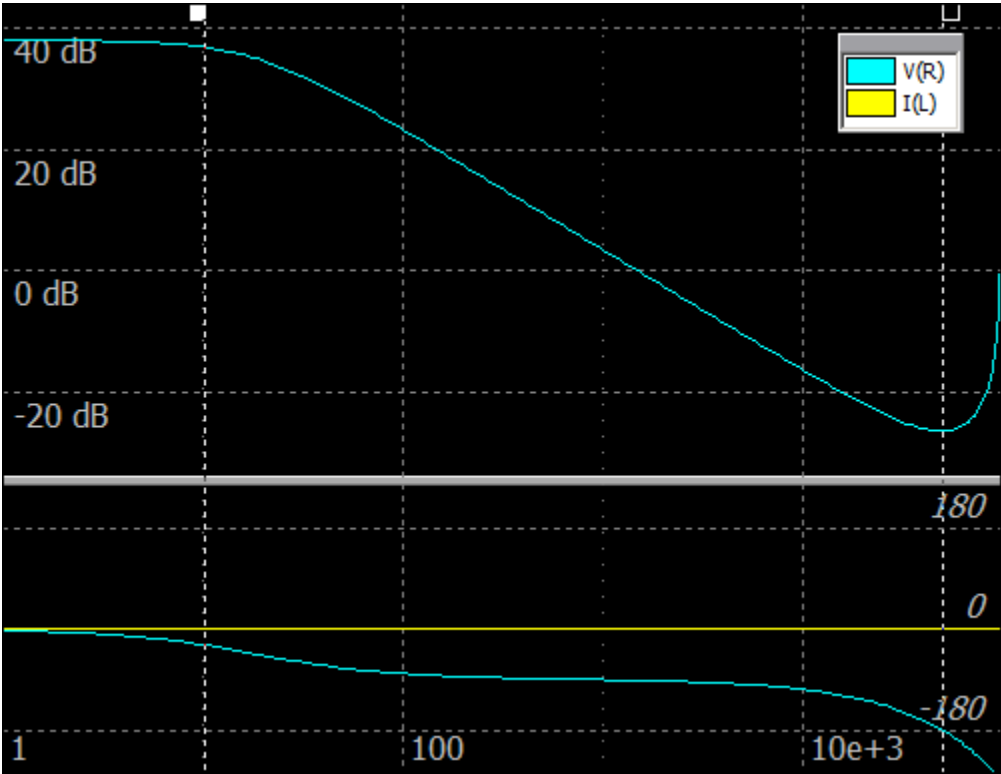
"Z-transform" method

Integration option can be used for some types of circuits and signals, where sampling option does not provide reliable results; in particular: converters working in DCM (discontinuous current mode), signals with wide spectrum (for example, pulsing signals), etc.

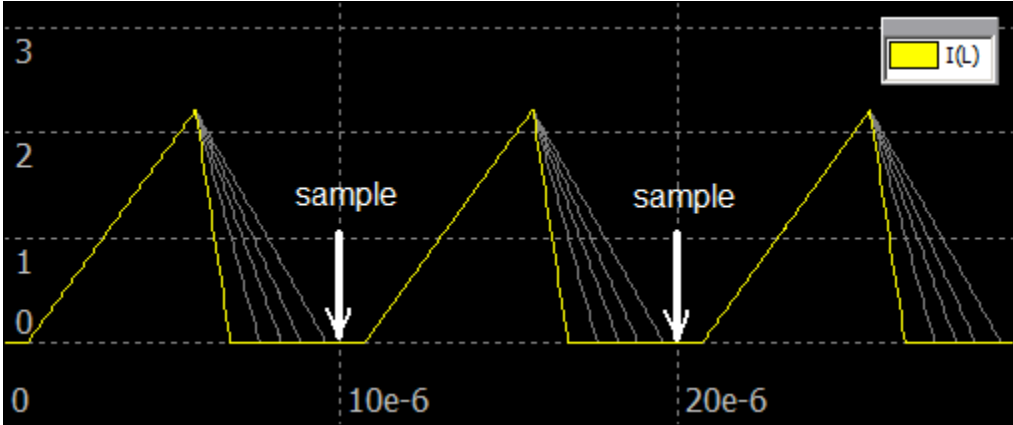
Here is an example of simple boost-converter:



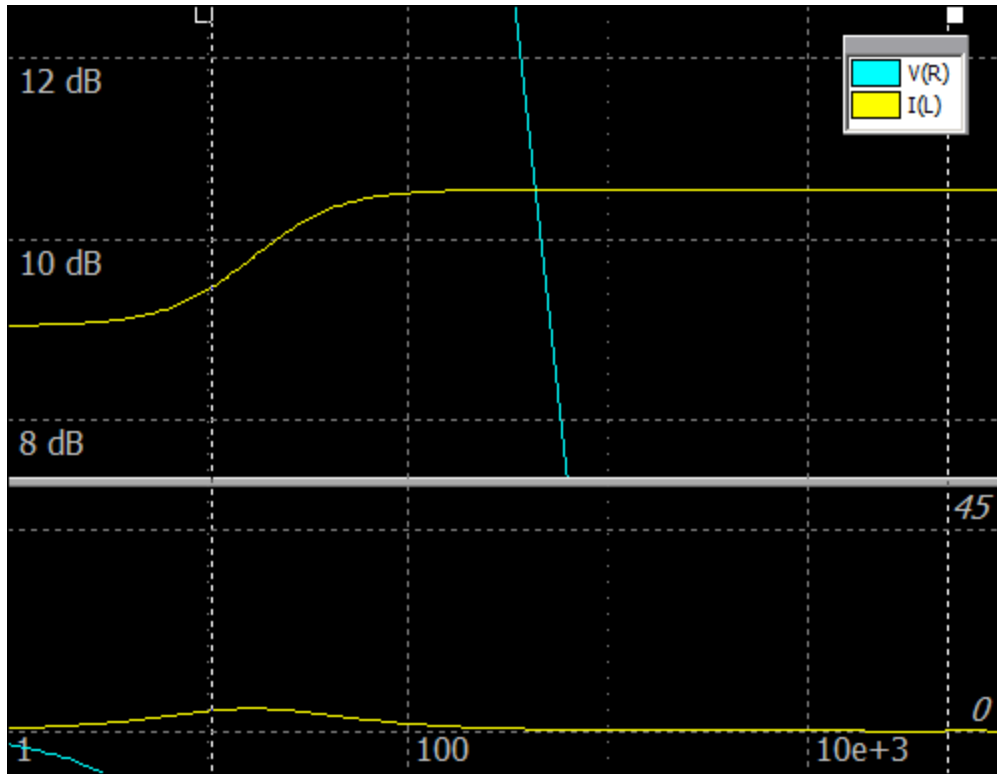
Here is AC response of voltage at the load R and current through the inductor L obtained with **sampling** “data collection” option:



While V(R) looks reasonable, I(L) is always zero with zero phase, which is definitely wrong. The reason is that the current through the inductor has been sampled at the moments when it is equal to zero, and all the variations of the current due to “disturbances” cannot be detected:

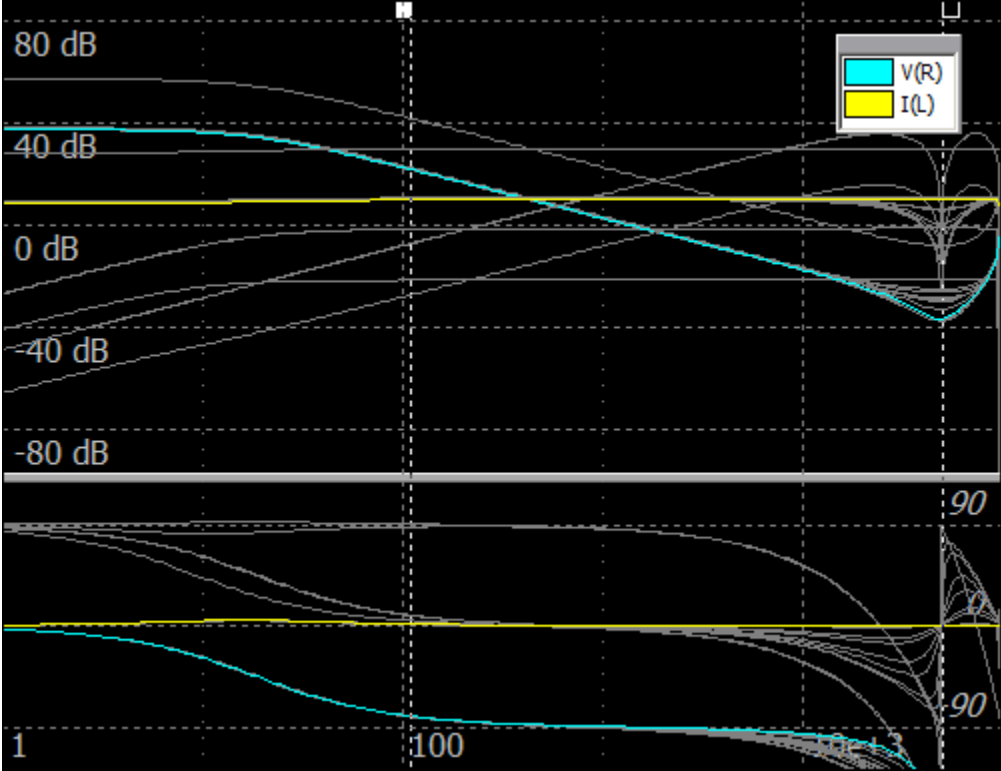


This can be fixed by using **integration** option: instead of sampling signal values at some discrete points, the signals are being integrated (averaged) during whole switching period. Here is a resulting characteristic with **integration** option, which shows non-zero current:

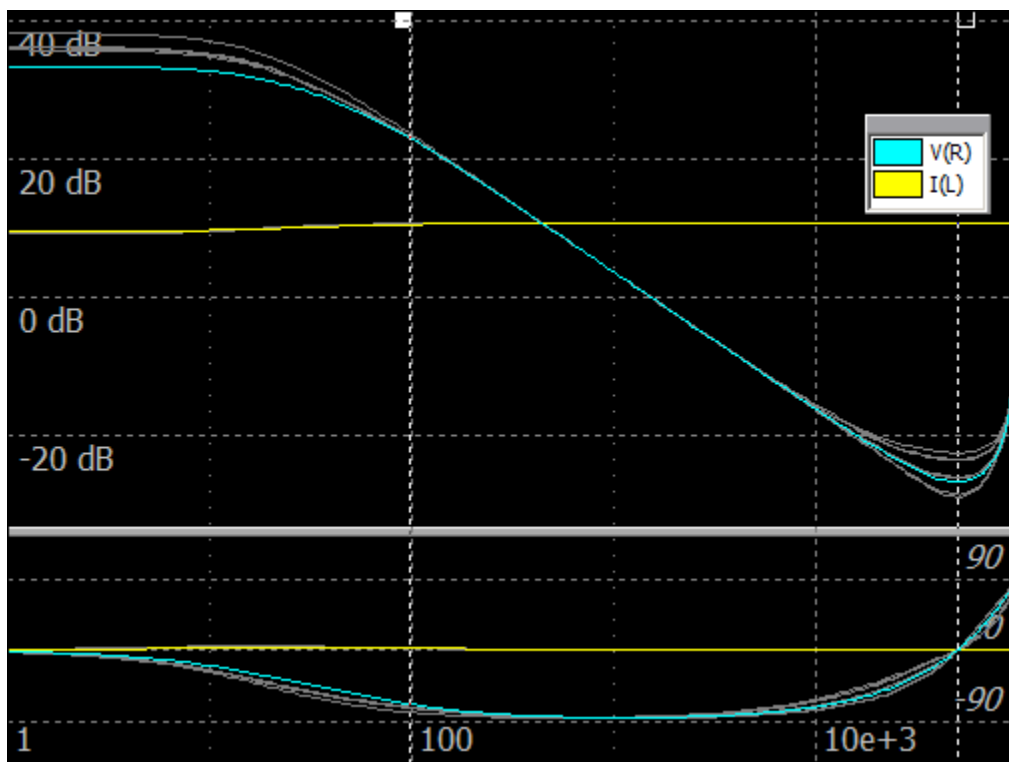


Integration option is slower than sampling, with higher phase error at high frequencies (close to Nyquist frequency).

Delayed integration is a modification of integration option, which eliminates yet another problem with DCM converters. It has been discovered that simulation results may depend on the time within switching period when “disturbance” is applied to the circuit. Here is a snapshot of many runs of Z-transform with different phase of “disturbance” relative to switching period, producing non-consistent results:



Delayed integration option waits for one switching period before starting integration after “disturbance” was applied to the circuit: it allows the circuit to get to proper operation mode after “disturbance”. Here are results of simulation with delayed integration mode:



The option gives consistent results, however the simulation is slower than other options, and produces higher phase error at high frequencies (close to Nyquist frequency).

A recommended strategy of analysis would be:

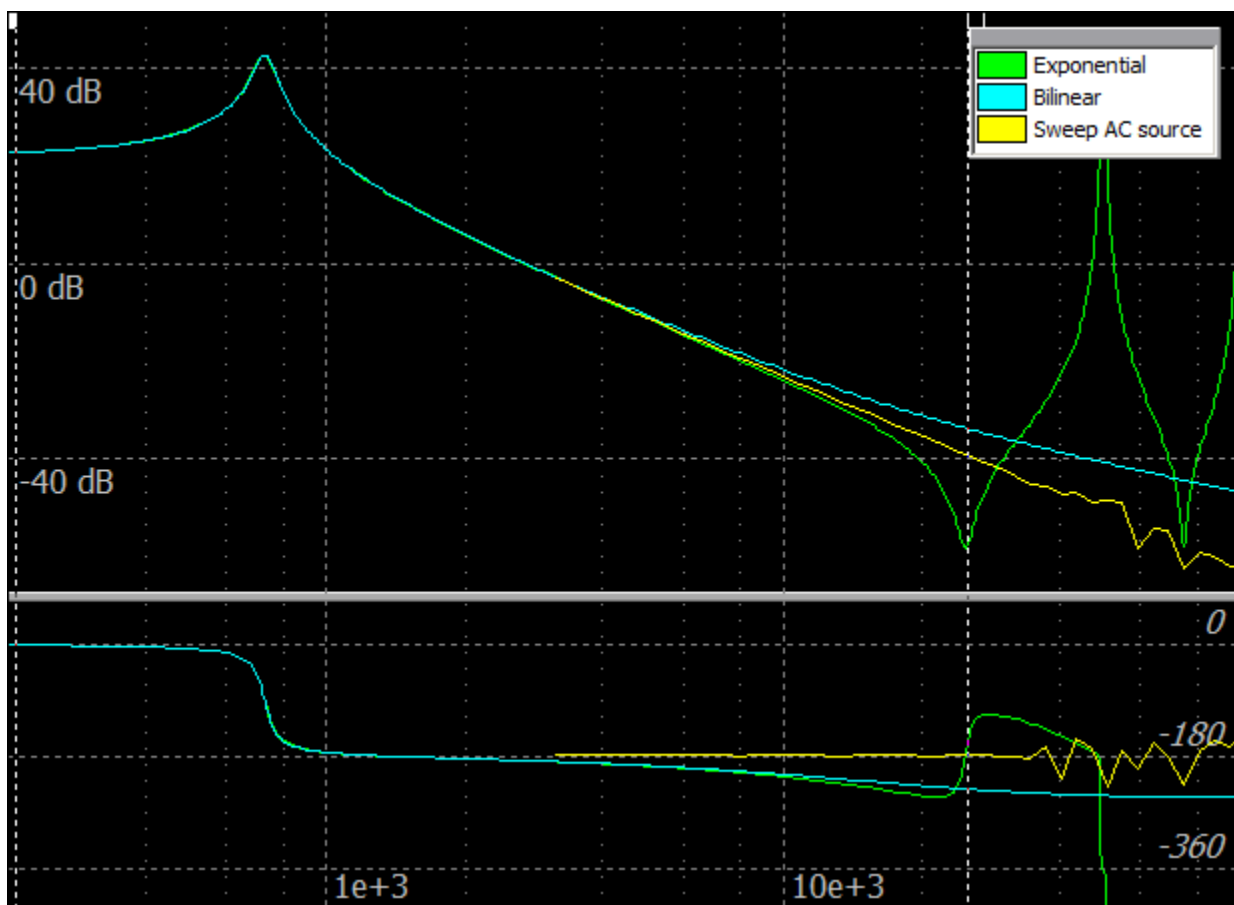
- Use **sampling** option as much as possible, since it is fastest and most accurate;
- If valid result cannot be obtained with sampling option, try **delayed integration**, in order to estimate how correct result should look like, even if it is not very accurate;
- Then use **integration** method with proper selected phase (delay) of switching period, in order to obtain valid results with reasonable accuracy.

“Z to s conversion” parameter.

Since Z-transform transfer function has been obtained from discrete time samples with period T , the transfer function is periodical with the period $1/T$, and only $0 \dots 1/2T$ frequency interval contains meaningful information. NL5 provides two options of presenting the data in a way most convenient to the user:

- **Exponential.** A standard substituting $z = e^{sT}$ is used. A resulting transfer function is periodical with period T .
- **Bilinear.** A bilinear conversion $z = (2+sT)/(2-sT)$ is used. A resulting transfer function is being “stretched” at high frequencies range, so that results obtained for Nyquist frequency ($z=-1$) are shown at infinite frequency. This makes transfer function look more familiar, without the periodical “fence” of spikes.

Here is how the characteristic of buck-converter looks like with different z to s conversion methods comparing to “Sweep AC source” (insertion) method:



Known issues.

NL5 rev 1.91 may not handle properly circuits with:

- Capacitors connected in parallel;
- Capacitors in parallel with voltage source;
- Inductors connected in series;
- Inductors in series with current source.

In this case NL5 will stop simulation with error message:

“Z-transform error: voltage/inductor loop. This error will be fixed soon”.

This error can be avoided by minor modification of the circuit: set small resistor in series with problematic capacitors, and big resistor in parallel to problematic inductor.